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A Regulated Control System for a Large Electromagnet

by

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Abstract: The design and performance of a regulated control system for the large electromagnet of the Laboratory is described. The system is an automatically self-correcting device to maintain the magnet current at any desired level. It is designed as a servomechanism to function as a follow-up device for varying input commands, as well as to compensate for certain extraneously caused current fluctuations. The latter include variations due to temperature changes, generator brush noise, generator structural asymmetries, and changes in generator speed caused by variations in line voltage. These fluctuations are randomly distributed in time, ranging from occasional small shifts and slow drift in the d.c. value of the current to noise with appreciable components up to 12 kc/sec. The present control system is designed to compensate for the drift and small shifts in the d.c. value of the current at any desired current level, but does not compensate for the components of noise of higher frequency. That is, it essentially determines the set point or d.c. value of the magnet current. Slow thermal drift and small shifts in the d.c. value are held to within one part in 10^5.

Future refinement of the current regulating system calls for an additional channel whose function is to compensate for the remaining noise

fluctuations. This can be accomplished by one of several methods, any one of which can be expected to achieve an overall precision of regulation of the order of one part in 10^5. For present uses of the magnet the existing one channel control system seems adequate, but future experiments will undoubtedly require addition of the second channel.

Features of the present control system include continuous variability of the magnet current from maximum negative to maximum positive values with stabilization provided at any desired level of the current, quick reversibility of the polarity for convenience in demagnetization, and complete elimination of switching in the magnet circuit itself.

Introduction

In a number of physical investigations a precisely controlled magnetic field is required. This field is usually obtained from an electromagnet, and therefore the problem of controlling the magnetic field becomes essentially the problem of controlling the current in the magnet coils.

The possible methods of control which may be used depend on the impedance level and power level of the magnet, the desired precision of control, and the desired speed of response. In low-current, low-power magnets thermionic regulation can be conveniently and effectively used, exciting the magnet coils directly from a hard-tube feedback amplifier.[1, 2] In high-power applications, however, the magnet current cannot be conveniently or economically supplied by vacuum tube sources, and power must be supplied by battery, motor generator or thyratron power supply. The initial expense, continuing maintenance, and great bulk of batteries limit their desirability as a source of power. Thyratron

1) M. E. Packard, Rev. Sci. Instr. 19, 435 (1948).

2) D. S. Hughes, W. L. Pondrom, G. B. Thurston, Phys. Rev. 75, 647 (1948).

power supplies possess the disadvantage of high ripple and noise content in their outputs, causing fluctuations in the magnet current, and tending to block the input stage of the feedback amplifier, limiting the possible gain and hence the attainable precision of stabilization. Generators also possess high output noise, but are simpler than thyratron supplies and more convenient to apply.

In the present application it was desired to stabilize the magnetic field of a 16 kw, 400 amp. laboratory electromagnet excited by a motor generator source. It was required that the field be continuously variable from maximum negative to maximum positive values, and that it be maintained constant at any particular setting to within one part in 10^5 for slow drift and to within 0.01 percent of the d.c. value of the magnet current for rapid fluctuations. Basically, the problem was to devise a high-gain, wide-band feedback control system which would respond satisfactorily to the desired inputs and neutralize output disturbances to the limits specified. The problem was complicated by the fact that a feedback loop enclosing large inductances and low resistances has a long time constant and therefore narrow bandwidth.

At the time of the initial designing of the system and during its construction, the more important of the two stability requirements above, so far as immediately prospective experimental research uses of the magnet indicated, was that the slow drift be stabilized. Accordingly, during design and refinement of the present system, attention was directed toward developing a system embodying the desired precision of stabilization of slow drift, sufficient rapidity of response to expected input programming without overshoot, continuous variability of current over the entire range, convenient and foolproof operation, and simplicity of maintenance. Although some preliminary work was done on the problem of eliminating small rapid fluctuations of the current, development of this part of the control system has mainly been left to a future date when experimental research using the magnet

demands greater overall precision of stabilization. For present uses of the magnet the existing one-channel system seems quite adequate. The magnet and its associated control system are used to facilitate research in the Ferromagnetics Section of the Laboratory for Insulation Research, and to date have been used in research during two doctorate thesis problems which have been completed and one which is currently in progress.

In the following section, methods for the stabilization of motor generator sources are discussed, noting their advantages and disadvantages with reference to the present problem. Succeeding sections are devoted to the analysis and design of the present system, analysis of its performance, and conclusions based on the study.

Requirements and Methods of Stabilization

General requirements. A stabilized control system for a laboratory electromagnet must perform two functions: it must first act as a follow-up system, responding accurately to the operator's command, and second it must act as a regulator, compensating for random current fluctuations. The nature of the operator's command or input signal may vary considerably depending upon the expected application of the magnet. For example, in the case of a cyclotron or mass spectrograph the program of input command consists simply of a constant value of current selected by the operator. That is, after the magnet current has reached the desired value, it remains constant for a long period of time. Conventional control systems developed for such applications are characterized by slow speed of response.[3,4] In other applications, such as the study of ferromagnetic resonance phenomena, the magnetic field may be required to change continuously or in a series of small steps. For both types of input the current

3) W. Y. Chang, Phys. Rev. 69, 60 (1946).

4) M. C. Henderson and M. G. White, Rev. Sci. Instr. 19, 19 (1938).

is required to follow the input command in a limited time. Speed of response of the control system must therefore be faster than for the cyclotron magnet.

Output current fluctuations which must be compensated by the stabilizer are of two kinds: (1) Relatively slow current variations due to temperature changes in the magnet and generator circuits, and slow changes in the line voltage; (2) Rapid current variations due to generator brush noise, to asymmetries of the generator armature and armature slots, to changes in the position of the armature with respect to the stator, to asymmetries in the field windings, or to sudden changes in the supply line-voltage.

With the magnet current held absolutely constant, the magnetic field of the magnet may still vary due to changes in the permeability and changes in the physical dimensions of the iron. In normal operating ranges of temperature, however, such field fluctuations are less than 0.01 percent of the field. If the required precision in a particular application does not exceed this limit, it is sufficient simply to stabilize the magnet current. If additional precision is required, it becomes necessary to monitor the magnetic field itself rather than the current in the magnet coils.

To fulfill simultaneously the functions of follow-up system and stabilizer, the control system must incorporate some means of measuring the instantaneous error between the input command and the actual value of the controlled quantity, in this case either the magnetic field or the magnet current, and to modify the output in such a manner as to minimize the error or reduce it to zero. In other words, some sort of feedback control system, or closed-loop follow-up system, is required. The problem of designing an appropriate system is essentially the problem of overcoming the large time constants introduced into the loop by the generator and magnet. Presence of large time lags in a closed-loop system causes slow speed of response and tends to produce instability or hunting. A

system with large time lag can respond accurately only to very slow changes in input command, and cannot compensate for rapid current fluctuations.

Dual-loop feedback methods. To overcome the effects of large time constants, systems employing two feedback loops have been developed. One loop of the control system responds to magnet current changes and is slow in speed of response. The second loop responds to voltage fluctuations and is relatively fast in response, compensating for voltage variations before they have time to cause sensible change in the magnet current. In a sense, the latter loop "anticipates" the current changes and tends to prevent them before they occur.

One of the first dual-loop systems described in the literature was designed by Anderson.[5)] It consisted of a vacuum-tube voltage regulator to maintain the generator voltage constant and a current-actuated element arranged to alter the operating point of the voltage regulator to compensate for slow changes in the current and to produce the desired d. c. magnet current. The system was used to control a 100 kw cyclotron magnet and maintained the magnet field constant to within 0.1 percent for periods of several hours. The voltage regulator portion of the system was a d. c. feedback amplifier similar to regulators described by Verman[6)] and Kilpatrick[7)], except that in Anderson's system the load disturbances were much smaller and the regulator was required to be inherently stable for long periods of time. The input to the voltage regulator was a voltage proportional to the voltage across the magnet. The voltage regulator controlled the current in the excitation generator field by varying the plate resistance of vacuum tubes in series with the field. The current-sensitive portion of the controller consisted of a Leeds and Northrup potentiometer-type recorder which

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5) H. L. Anderson, J. R. Dunning, and D. P. Mitchell, Rev. Sci. Instr. 8, 497 (1937).

6) L. C. Verman, and H. J. Reich, Proc. I.R.E. 17, 2075 (1929).

7) F. E. Kilpatrick and C. P. Bernhardt, Electronics 7, 352 (1934).

measured the difference between a reference voltage and the voltage across the magnet, and actuated a motor-driven potentiometer to adjust the operating point of the voltage regulator. The response time of the recorder was long so that it did not respond to rapid changes in the current, but served simply to set the d.c. level. Slow drift was less than 0.01 percent over periods of many hours.

Since this type of controller can respond to only very slow variations of input signal, its application is excluded in the present problem.

Another example of the two-loop feedback control system is the cyclotron stabilizer and controller described by Henderson.[4)] The design of the circuit was based on a paper by Gilbert[8)] and utilized, as the current-sensitive element, a galvanometer actuating two photocells in the grid circuit of two parallel type 2A3 tubes, whose plate current determined the signal on the grids of a phase-controlled thyratron power supply which excited the generator field windings. Although we are not here primarily interested in thyratron systems because of their inherent high ripple and output noise, the above system is noteworthy because it utilized the principle of multiple loops in a feedback system containing regulated thyratrons in conjunction with the generator used to excite the electromagnet. The plate current of the 2A3's determined the inductance of a saturable reactor which in turn determined the phase of the signal voltage on the thyratron supplying the generator field current. The time constant of the current-stabilizer portion of the circuit was very large and it therefore responded only to very slow variations of input or output disturbances, as in the Anderson regulator. The sensitive element of the voltage-regulator loop was a shunt in series with the generator field windings. A separate winding on the saturable reactor was connected across this shunt in such a way that rapid variations in field current resulted in compensating changes of signal-voltage phase at the thyratron grids

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8) R. W. Gilbert, Proc. I.R.E. 24, 1239 (1936).

and thus compensated for the current fluctuation. The voltage regulator actually did not respond to output voltage variations, but since the time constant of the magnet was much larger than the time constant of the field circuit of the generator the field current appeared to be practically in phase with the generator voltage so far as the output current was concerned.

The band width of this controller is potentially larger than that of Anderson's system, since here the excitation generator together with its associated time constant is not present in the loop. However, this advantage is offset by the additional noise introduced into the circuit by the thyratrons themselves, even though the output of the thyratrons is passed through an L-C filter. Furthermore, the band width of the voltage regulator is limited by the time constant of the main generator field circuit. The gain permissible in the voltage regulator loop, and hence the stabilization ratio, is limited by the time constants of the field and of the thyratron supply to a fairly low value consistant with stability. A more recent application of a very similar system has been described by Chang.[3]

A circuit which, though basically similar in arrangement to Henderson's system, avoids the use of noise-producing thyratrons has been described by Lawson.[9] As in the previously described system, no excitation generator was used and the time constant associated with it was thus eliminated. In this case it was possible to excite the generator field windings directly from the 225 v d.c. mains, controlling the current in the windings by inserting eight parallel-connected type 2A3 tubes in series with the field. Again, as in the previous system, a galvanometer and photocell arrangement was used to actuate the current stabilizer loop, although in this case the photocell circuit was considerably more sensitive than in the former. An improvement over Henderson's

9) J. L. Lawson and A. W. Tyler, Rev. Sci. Instr. 10, 304 (1939).

system was accomplished in the voltage-regulator loop by the utilization of a modulated carrier in the amplifier circuit. By modulating an audio frequency signal with the voltage across the electromagnet, amplifying the modulated signal, demodulating and controlling the grids of the 2A3's with the demodulated signal it was possible to reduce the time constant associated with the amplifier. This regulator possessed the same sort of limitations as the foregoing systems, namely, slow response of the current stabilizer and limited precision due to the time constant of the generator field circuit contained as an element in the voltage regulator loop. The system was designed to maintain current stability to 1 part in 5000. Long period drift was held to within 1 part in 50,000 for periods of several hours. The current could be changed from one value to another in about 10 seconds, but the range variability was limited.

A recent system for stabilizing currents of up to 100 amp. in highly inductive loads such as electromagnets has been developed and described by Caro and Parry[10] which featured convenient control of current over a wide range (10 to 100 amp.) and utilized magnetic transductors connected as d.c. transformers as the sensitive element of the current stabilizer loop. The field of the main generator was excited directly by the output of the d.c. amplifier which controlled the current level according to the set point of the amplifier, and simultaneously functioned as a voltage regulator to compensate for rapid fluctuations of the current about the set point. This system offers several advantages over those previously mentioned. First, it contains no moving parts except the generator itself and therefore mechanical maintenance problems are reduced. Second, the system used to compare the magnet current with the standard is capable of considerable long-period accuracy. Furthermore, the system offers faster response than the foregoing. However, the generator field circuit again

10) D. E. Caro and J. K. Parry, J. Sci. Instr. 26, 374 (1949).

was included in both the current stabilizer loop and the voltage regulator loop, hence the bandwidth of the voltage regulator is once more limited. Also, with the magnetic-transductor current comparator the magnet current can be changed in discrete steps only and cannot be varied continuously from a maximum positive value to a maximum negative value. The system therefore does not lend itself to use in the present problem, even though the speed of response of the current stabilizer could probably be made sufficient to follow expected inputs. Caro's stabilizer reduced short period current variations to less than 1 part in 2000. Current changes due to load impedance variations were reduced by a factor of 20. This is not sufficient precision for the present application.

Parallel-connected dual-loop feedback method. The systems so far discussed possess one characteristic in common; that is, in each of them at least one large time constant-circuit element is common to both the current stabilizer and voltage regulator loops. Another property possessed by all of the foregoing described systems except the last is inherent slowness of response of the current stabilizer loop. In fact, each is operating as a regulator or is regulating against output disturbances only. Since the input is changed only at infrequent intervals, the regulator is not required to act as a follow-up system, or servomechanism, responding to varying input.

A system which eliminates all the large time constants from the voltage regulator loop has been designed and built by Sommers et al.[11] The system consisted of two loops or channels. The voltage regulator loop, or the high-frequency loop as it is termed in Sommers's paper, drove the magnet directly as a load, being connected in parallel with the low-frequency loop, or current-stabilizer loop, which contained the generator as an element. Parallel operation of the two loops was made possible by suitable shaping of the frequency characteristics of each of

11) H. S. Sommers, Jr., P. R. Weiss, and W. Halpern, Rev. Sci. Instr. 20, 244 (1949).

the two loops. To accomplish this result the response of the two loops together as a continuous function of frequency and the spectrum of the input signal and output disturbances had to be considered. The bandwidth of the low-frequency loop had to be wide enough so that the power demands on the voltage regulator loop could be met with a practical and economical hard-tube feedback amplifier.

Part of the amplification in each loop was provided by an amplifier common to both. The inputs for both loops were fed directly into this amplifier. The output of the amplifier was fed through a crossover filter which was designed to separate the frequency components into two bands appropriate to the response characteristics of each loop. In passing we may note that the use of a common amplifier, as in the above system, is not essential to the parallel-connection method; the method is equally applicable to cases which might require completely separate loops. A blocking reactor prevented the armature of the generator from effectively short circuiting the magnet so far as the high-frequency loop was concerned. The output of the high-frequency loop was coupled to the magnet through an RC coupling network. The bandwidth of the low-frequency loop was determined by the time constants of the generator field circuit and the magnet.

The system was used to regulate the current of a 40 kw electromagnet and reduced the noise output of the associated generator to 0.2 percent of the unregulated value. The d.c. drift after thermal equilibrium was reached was 4×10^{-3} parts per minute.

The design of a parallel-connected dual-feedback loop system is essentially a problem in designing a narrow bandwidth servomechanism operating both as a servomechanism and a regulator, and an accompanying dynamic filter operating as a voltage regulator to regulate against the higher frequencies of output voltage noise. This type of system eliminates the slow-response characteristic of the preceding types of systems and, due to the increased bandwidth and gain possible

in the high-frequency loop, increases the degree of precision of stabilization which is theoretically possible. The method thus holds promise for application in the present problem since here it is desired to design a servomechanism giving specified response to a varying input, while simultaneously subject to output disturbances whose effect must be reduced to a very low value.

Methods of monitoring the magnetic field. It is of interest at this point to mention briefly some of the methods which may be used to monitor the magnet field. With the current in the magnet windings constant, any fluctuations in the air-gap field are due to changes in the permeability and physical dimensions of the iron. Over the range of normal operating temperatures such variations are less than 0.01 percent of the field. Thus, if the degree of precision desired from the regulator permits, one may monitor the magnet current rather than the magnet field directly. Several methods of current monitoring have already been mentioned. The photocell-galvanometer-shunt method has the advantage of high sensitivity and absence of drift, but possesses the disadvantages of slow response time and difficulty in building a rugged optical system. The potentiometer type recorder-controller used by Anderson was an on-off type of device having a "dead space" of about 0.02 mv and thus the degree of precision of monitoring the current is limited. Also the response time of the device is slow. D.c. transformers such as those used by Caro are limited in sensitivity.

A variation of the galvanometer method converts the d.c. difference voltage between the drop across the shunt and a reference voltage into an a.c. voltage by means of a vibrator. The a.c. voltage is then amplified in a conventional a.c. amplifier and the output either used directly to drive a control motor appropriately placed in the circuit or rectified and applied to the current stabilizer loop in a suitable place. This method has the advantages of high sensitivity, fast response, minimum drift, and simple and rugged construction.

In applications requiring greater precision of regulation the magnet field itself must be monitored and any of several methods may be used. A fluxmeter may be placed in the magnet gap and its output used to actuate photocells in the current stabilizer circuit.[12)] A limitation of this method is its tendency to drift. A vacuum tube may be used as a magnetron in the magnet gap, but its usefulness is limited by its instability.[13)] The change of resistance of bismuth in a magnetic field might be utilized, but here difficulty is encountered in maintaining the temperature of the sensitive elements constant enough for high precision.[14)] The nuclear resonance phenomenon may be used to produce a signal.[1)] The range of control of this method is somewhat limited, but it does have the advantage of high sensitivity. Finally, a rotating coil fluxmeter may be used. In this method a coil is rotated at a precise speed in the air gap, producing a voltage whose magnitude is directly proportional to the flux density in the gap. The method is convenient and precise, but has the possible disadvantage of taking up space in the air gap and thus interfering with research equipment mounted therein.

Performance Requirements

As previously mentioned in the introduction, it is necessary (1) that the field of the magnet be stabilized at any desired level to within one part in 10^5 for slow drift (and ultimately to within one part in 10^5 for rapid fluctuations when future experiments demand the increased overall precision of regulation), (2) that the field be continuously variable from negative to positive values, and (3) that the speed of response of the controlling system be fast enough to follow expected input variations. The input variations are expected to be of two types; namely, monotonically changing at a slow rate, or increasing or decreasing in a series

12) C. E. Wynn-Williams, Proc. Roy. Soc. London A145, 250 (1934).
13) A. O. Nier, Rev. Sci. Instr. 6, 254 (1935).
14) G. S. Smith, Elec. Eng. 56, 441 (1937).

of small steps which occur at intervals of about 1/2 minute or longer. For the latter type of input the response of the system must have no overshoot, and must respond fast enough so that sufficient time is available after the occurence of a step for the current to approach a steady value and for data readings to be taken before the next step occurs. This implies that the effective time constant of the system must be of the order of a few seconds at most. For monotonically changing input the rate of change of current to be expected is of the order of 5 to 10 amp./sec. It is required that the controller follow an input of this type with no sensible error. It can be shown that a servomechanism whose regulatory behavior meets the specifications on precision of regulation will also satisfy the last requirement.

In addition to the above performance requirements, it is desired that the system be simple to operate, require a minimum of adjustment and maintenance and be economical to build.

Synthesis of the system is accomplished in eight principal steps:

1. Selection, in general terms, of a proposed preliminary design.
2. Choice of the major system components.
3. Derivation of the system differential equations, and determination of the transfer functions of the various components.
4. Determination of major time constants and rough estimation of overall loop response.
5. Determination of the output to input response of minor loops, and calculation of minor loop gain.
6. Determination of overall transfer function of major loop, loop gain, and output response characteristic.
7. Comparison of the calculated loop response with the required response, and modification of the system design as required.
8. Construction of the system, measurement of response, and comparison of measured results with the calculated predictions, and, if the two results are not in agreement, analysis of causes and performance of necessary modifications.

General Description of the Design of the Present System

A parallel-connected dual-loop design was chosen as a system which meets the specified requirements. The low-frequency loop, or current stabilizer, was designed to satisfy the input response specifications, and the two loops together designed to give the proper degree of stabilization.

A high-frequency loop consisting of a direct coupled feedback amplifier or dynamic filter was designed and built in a preliminary form. It was designed to have a bandpass from 1 cps to 12 kc/s, and to drive the magnet in parallel with the generator. However, since the experimental uses of the magnet to date have not required the elimination of the small, rapid fluctuations of the noise, attention has been directed toward refining and completing the low-frequency loop of the system, and the high-frequency loop has not yet been incorporated. The latter may be added at any time that the experimental applications of the magnet demand the additional overall precision of regulation, and may consist either of a dynamic filter as preliminarily designed, or possibly of small auxilliary coils at the magnet air gap, driven electronically in accordance with an error-measuring device placed in the gap itself. The latter method offers the advantage of better decoupling from the low-frequency loop and lends itself to the use of a rotating-coil gaussmeter as the sensing element, or to a nuclear resonance sensing element. Either method can be expected to achieve an overall precision of regulation of the order of one part in 10^5 when properly matched frequency-response-wise to the low-frequency channel.

The preliminary design of the high-frequency loop utilized a direct-coupled feedback amplifier whose output stage drove the magnet. A d.c. amplifier was used in order to extend the low-frequency response of the loop down to the cut-off frequency of the current stabilizer loop. The shape of the transfer function of the loop was determined by passive networks, and therefore changes in

amplifier characteristics due to tube aging and similar causes were minimized. The feedback signal for the loop was taken from the voltage across the magnet. An inductor between the magnet and the generator armature served to isolate the armature from the output of the voltage regulator or dynamic filter.

The specified precision of regulation makes it permissible to monitor the current of the magnet windings rather than the air-gap magnetic field directly. The selection of a modulated carrier system for the low-frequency loop permits greater bandwidth, eliminates grounding difficulties inherent in d. c. systems, permits greater gain in the loop, and simplifies circuitry by allowing the use of an a. c. voltage amplifier.

With this design the d. c. voltage drop across a monitoring shunt in series with the magnet coils is compared with a reference voltage, and the difference converted to a. c. by vibrator and amplified. The amplified voltage is used to excite a two-phase, low-inertia servo control motor. The control motor actuates potentiometers in the generator-field circuit in such a way as to cause the magnet current to follow the input signal, or, in other words, to reduce the error. Power for the field is supplied by a separate d. c. generator through the potentiometers connected in a bridge circuit across the field. This permits continuous variation of the field current, and hence of the magnet current.

The speed of response and bandwidth of the loop depend on the time constants associated with the amplifier, the control-motor-potentiometer unit, the generator and the magnet. Of these, the time constant of the generator and magnet predominate. The bandwidth of the current stabilizer loop is designed to be wide enough so that a comparatively low-power feedback amplifier in the voltage regulator loop can regulate the noise components in the remaining frequency range over which the noise must be regulated.

Design of the Low-Frequency Loop

The error-measuring circuit. The schematic diagram of the comparator or error-measuring element is shown in Fig. 1, and its position in the control system is indicated in the block diagrams of Figs. 2 and 3. The variable resistance R may be expressed* as

$$R = R_1 \left| \frac{\beta}{\beta_m} \right| \qquad . \tag{1}$$

where β = angular position of the potentiometer contact in degrees; and β_m = maximum positive or negative angular position of the potentiometer contact = $\pm$ 2700°.

Let $R_T = R_3 + R_4$, then R_3 and R_4 may be expressed by

$$R_3 = \frac{R_T}{2}\left(1 - \left|\frac{\beta}{\beta_m}\right|\right) \qquad , \tag{2}$$

$$R_4 = R_T - R_3 = \frac{R_T}{2}\left(1 + \left|\frac{\beta}{\beta_m}\right|\right) \qquad . \tag{3}$$

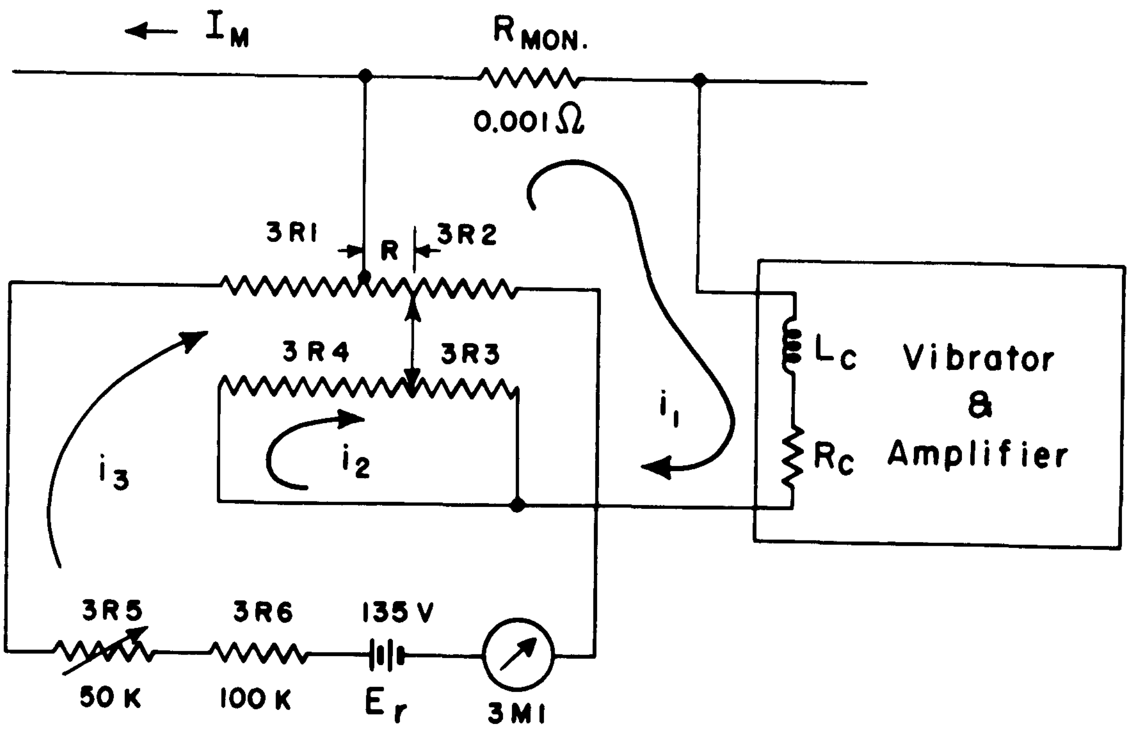

Fig. 1. The error-measuring circuit.

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*The unit prefix number of the various resistors is omitted in the following derivation, i.e., $3R_1$ is written simply as R_1, etc.

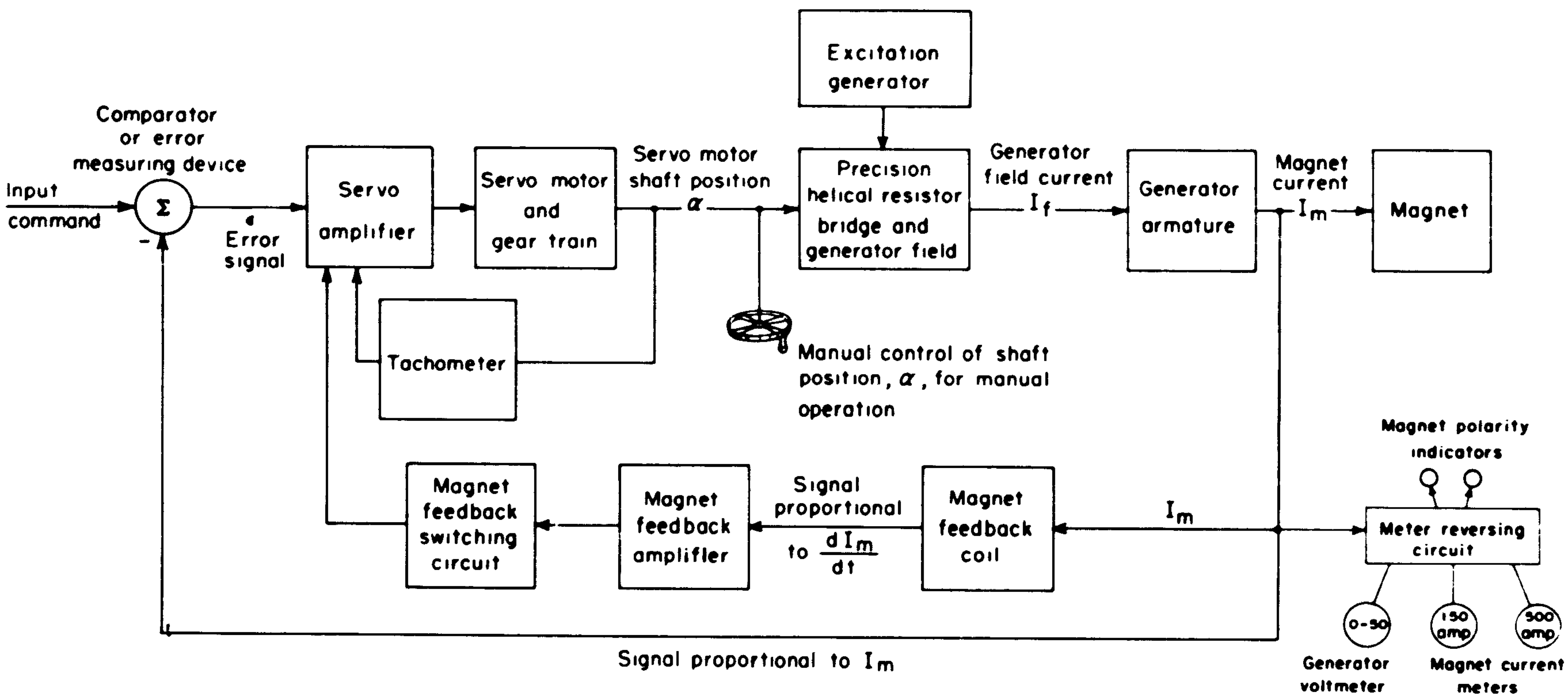


Fig. 2. Simplified block diagram of the magnet-control system.

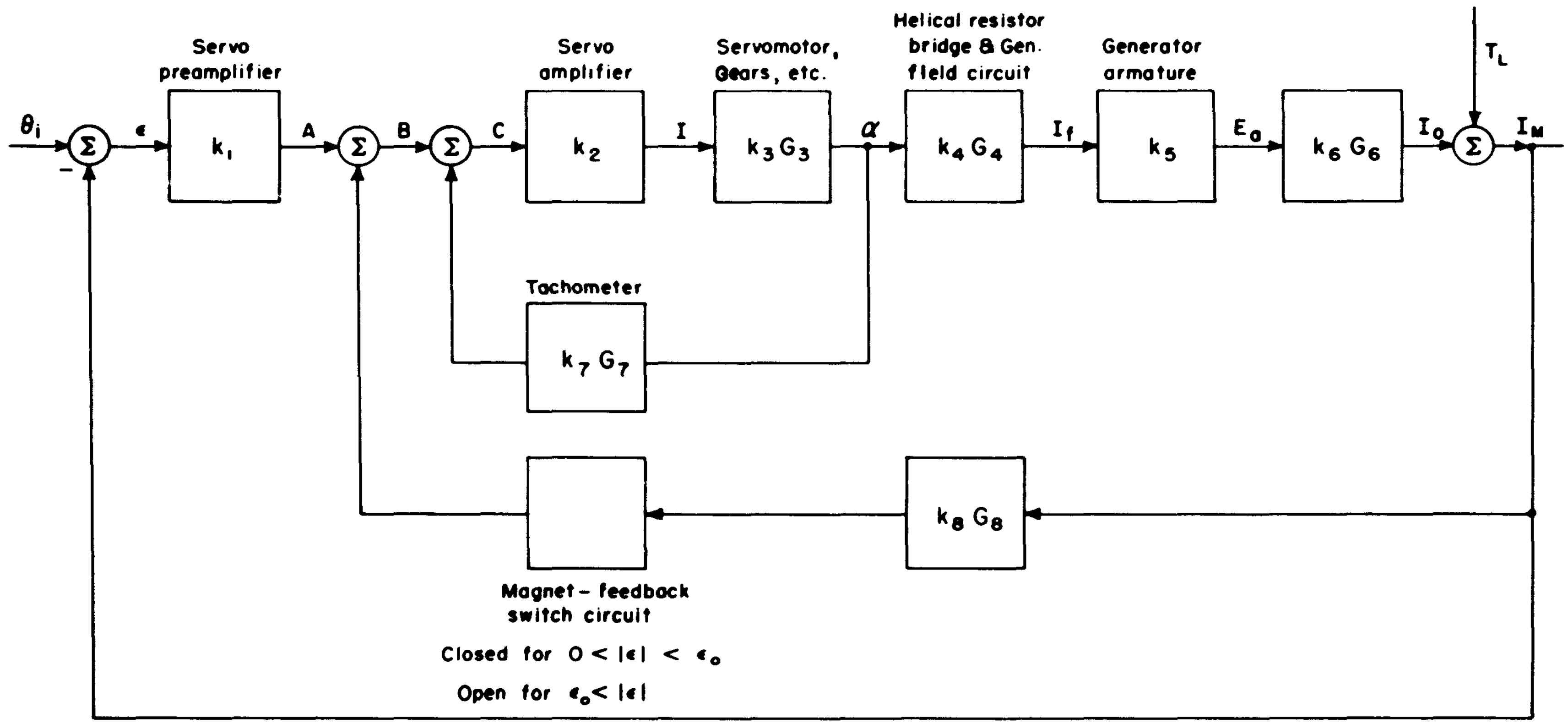


Fig. 3. Block diagram of the low-frequency loop. $k_3G_3 = k_3/s(\tau_3 s + 1)$; $k_4G_4 = k_4/(\tau_4 s + 1)$; $k_6G_6 = k_6/(\tau_6 s + 1)$; $k_7G_7 = k_7 s$; $k_8G_8 \simeq k_8 s$.

The loop transform equations for the circuit are

$$\left.\begin{aligned} \left[(R_{mon}+R_c+R_3+R)+sL_c\right]i_1(s)-R_3\,i_2(s)-R(s)\,i_3(s)-R_{mon}I_M(s)&=0\\ -R_3\,i_1(s)+R_T\,i_2(s)+0+0&=0\\ -R(s)\,i_1(s)+0+(2R_1+R_5+R_6)\,i_3(s)+0&=E_r(s) \end{aligned}\right\}\quad(4)$$

Solution of Eqs. (4) may be simplified by making certain approximations. R_3 and R_4 may be replaced by an equivalent series resistance R_{eq} in loop 1, and loop 2 therefore will be eliminated.

$$R_{eq}=\frac{R_3R_4}{R_3+R_4}=\frac{\frac{R_T}{2}\left(1-\left|\frac{\beta}{\beta_m}\right|\right)\frac{R_T}{2}\left(1+\left|\frac{\beta}{\beta_m}\right|\right)}{\frac{R_T}{2}\left(1-\left|\frac{\beta}{\beta_m}\right|\right)+\frac{R_T}{2}\left(1+\left|\frac{\beta}{\beta_m}\right|\right)}\quad. \qquad (5)$$

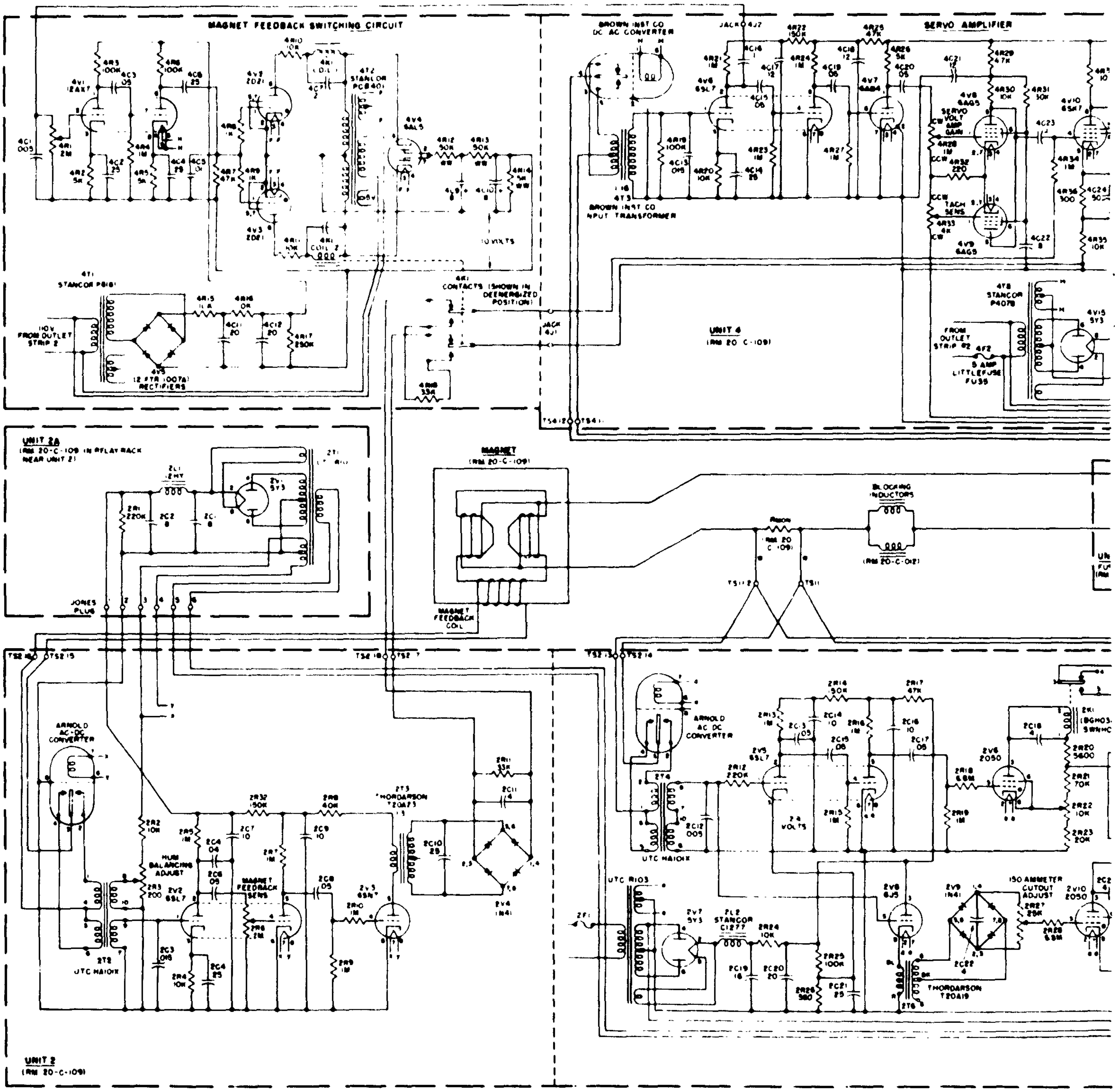
The potentiometer R_3, R_4 is introduced into the circuit to maintain the resistance across the input to the converter and amplifier more nearly constant. The sum $R+R_{eq}=R_o$ varies from $500\,\Omega$ at $\left|\frac{\beta}{\beta_m}\right|=0$ to $625\,\Omega$ at $\left|\frac{\beta}{\beta_m}\right|=1/2$ and back to $500\,\Omega$ at $\left|\frac{\beta}{\beta_m}\right|=1$. The gain of the amplifier changes with changes in the input resistance, and it is therefore desirable to maintain the input resistance approximately constant. The above variation of R_o over a range of about $\pm$ 12 percent is satisfactory in this respect.

Since $R_5+R_6 \gg R$ and the maximum voltage drop across R due to $i_1 < 0.5$ v, variation of R from zero to R_1 has negligible effect on i_3, and i_3 may be considered practically constant; hence we may eliminate the loop equation for loop 3. Equations (4) therefore reduce to one equation,

$$(R_{mon}+R_c+R_o+sL_c)\,i_1(s)-R(s)\,i_3(s)-R_{mon}I_M(s)=0\quad. \qquad (6)$$

$R_o \gg (R_{mon}+R_c)$, so we may neglect $(R_{mon}+R_c)$. In the frequency range of interest, 0 to 20 cps, which is the bandwidth of the servo amplifier itself, the term $sL_c \ll R_o$. Also, to a close enough approximation, i_3 is a constant equal to $E_r/(2R_1+R_5+R_6)$; therefore Eq. (6) may be written

$$R_{mon}I_M(s)+i_3R(s)=R_o\,i_1(s)\quad. \qquad (7)$$


MAGNET FEEDBACK SWITCHING CIRCUIT
SERVO AMPLIFIER
BROWN INST CO DC AC CONVERTER
BROWN INST CO INPUT TRANSFORMER
4R1 CONTACTS (SHOWN IN DEENERGIZED POSITION)
UNIT 4 (RM 20-C-109)
UNIT 2A (RM 20-C-109 IN RELAY RACK NEAR UNIT 2)
MAGNET (RM 20-C-109)
MAGNET FEEDBACK COIL
BLOCKING INDUCTORS
JONES PLUG
ARNOLD AC-DC CONVERTER
HUM BALANCING ADJUST
UTC HA101X
UNIT 2 (RM 20-C-109)

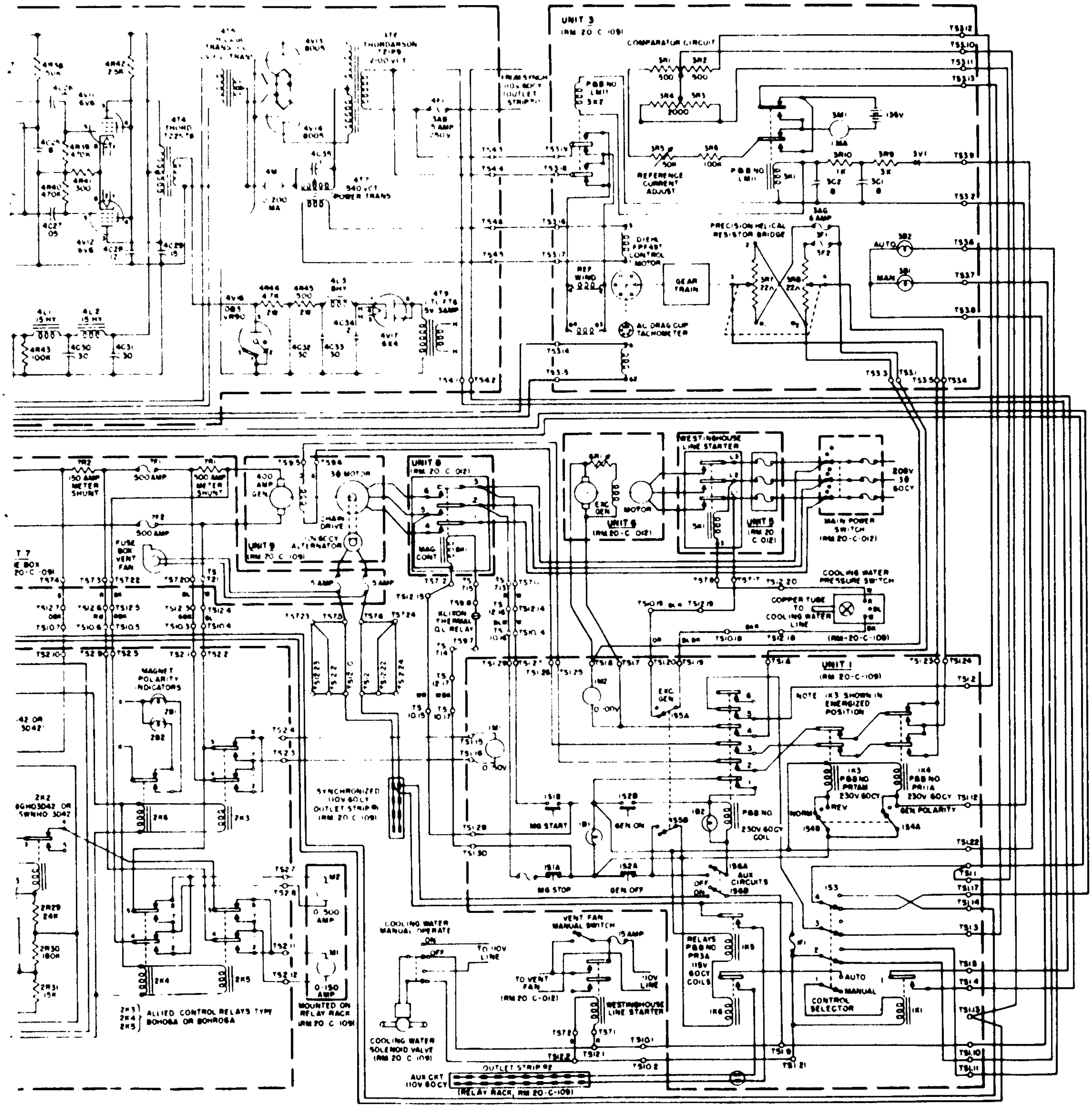


FIG. 9 SCHEMATIC DIAGRAM OF MAGNET CONTROL SYSTEM

Equation (7) is of the form

$$\theta_s(s) - \theta_i(s) = \epsilon(s) \qquad , \qquad (8)$$

where $\theta_o(s) = R_{mon}\, I_M(s)$; $\theta_i(s) = -i_3R(s) = -\dfrac{i_3R_1}{|\beta_m|}\beta(s)$; $\epsilon(s) = +R_o i_1(s)$. It should be noted that the input signal $\theta_i(s)$ is determined solely by the angular position $\beta(s)$ of the control potentiometer R_1, R_2 as desired.

<u>The AC-DC converter, servo amplifier, and servo control motor</u>. The first two blocks in the low-frequency loop are the converter and servo amplifier and the servo motor (units 4 and 3 in the schematic diagram, Fig. 9, at the middle of this report). This portion of the system forms a minor loop consisting of a servo amplifier driving a two-phase control motor in the forward path, and tachometric feedback in the feedback path of the loop. The converter consists of a vibrator and input transformer circuit which converts the d.c. error signal into an amplitude modulated a.c. signal whose carrier frequency is 60 cps. In the servo amplifier the a.c. signal is amplified through three stages of amplification in 4V6a, 4V6b, and 4V7. In the following stage the amplified error signal voltage and the feedback voltage from the a.c. tachometer attached to the servomotor shaft are combined algebraically in the common plate load resistor, 4R30, of 4V8 and 4V9. The error signal voltage is applied to the grid of 4V8 through the servo voltage-amplifier gain potentiometer, 4R28; and the tachometer feedback voltage is applied to the grid of 4V9 through the tachometer sensitivity potentiometer, 4R33. The first of these controls the forward gain of the servo amplifier and the latter determines the amount of tachometric feedback. At this stage in the amplifier the magnet-feedback voltage is applied to the grid circuit of 4V10 so that the voltage appearing between grid and cathode is the sum of the magnet-feedback voltage and the voltage from 4R30. The former is a more or less slowly varying d.c. voltage whose amplitude depends on the time rate-of-change of the magnet current, while the latter is a 60 cps voltage modulated by

the error signal and the tachometer signal. Thus, the magnet-feedback voltage may be viewed as a varying bias voltage on 4V10 and the voltage from 4R30 as the signal voltage. As the magnet-feedback voltage increases, the gain through 4V10 correspondingly decreases. 4V10 functions as a split-load phase inverter, and its output is applied to 4V11 and 4V12 in push-pull. These in turn drive the output or power stage (4V13 and 4V14) of the servo amplifier.

Design of the power stage of the amplifier depends on the characteristics of the servo-control motor to be used in the system, and it is therefore of interest at this point to direct our attention briefly to the design considerations involving the servo motor before discussing the design of the output stage of the servo amplifier.

To select a suitable control motor it is first necessary to determine the inertia and friction of the motor with its associated load, then to select a motor whose torque and speed characteristics give the desired acceleration. In the present case, the load consists of a helical resistor bridge and its associated gear train. This is shown schematically in Figs. 4 and 5; in the latter figure the constants are referred to the output shaft of the control motor.

Two-phase servo control motors are designed to give about 63 percent of rated locked torque at 55 percent of synchronous speed. Over this range of speed, from 0 to 55 percent, the speed-torque characteristics are approximately linear, and the motor behaves like a viscous damped device with very low friction.

The L-transform equations of motion for the motor with load are

$$T(s) = k_1 I(s) = J_o s^2 \alpha(s) + s f_o \alpha(s) \quad , \qquad (9)$$

where T(s) is the Laplace transform of the developed torque of the motor. Thus the transfer function for the motor with load is

$$\frac{\alpha(s)}{I(s)} = \frac{k/f_o}{s\left(\frac{J_o}{f_o} s + 1\right)} = \frac{k/f_o}{s(\tau_3 s + 1)} \quad . \qquad (10)$$

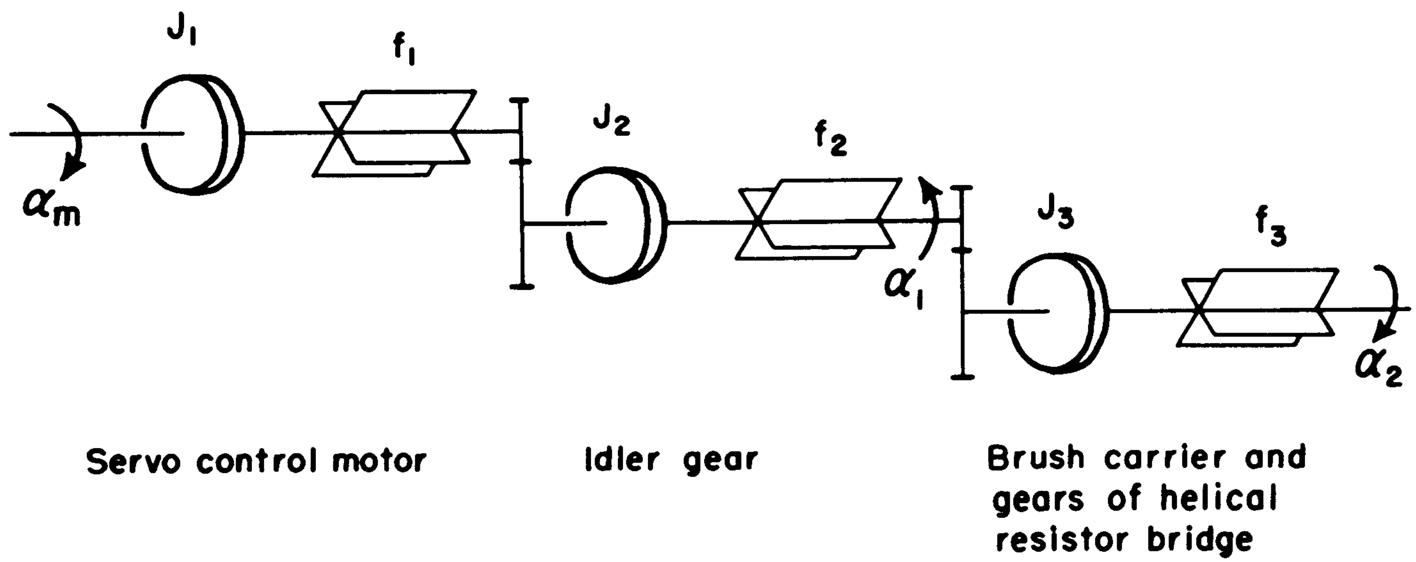


Fig. 4. Schematic diagram of the servo-control motor and its load. $r_1 = 3.2$; $r_2 = 4$; a_m = angular position of motor shaft.

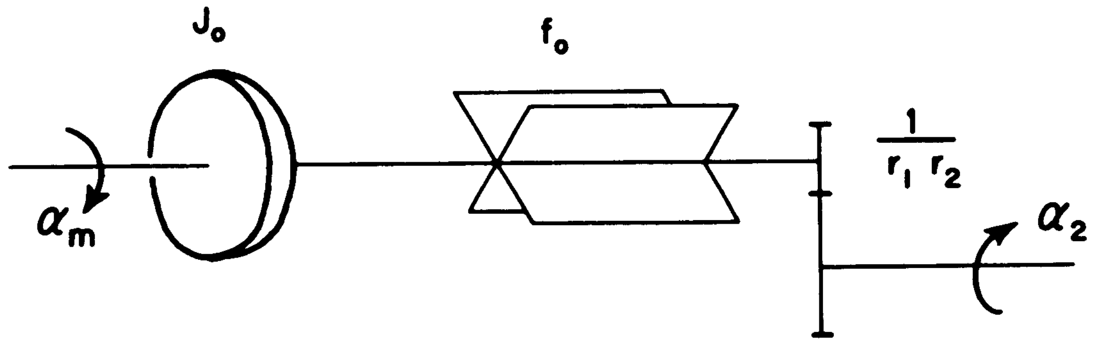


Fig. 5. Simplified diagram of the servo-control motor and its load, with all quantities referred to the control-motor shaft. $J_o = J_1 + \frac{1}{r_1^2}(J_2 - \frac{1}{r_2^2} J_3)$; $f_o = \frac{1}{r_1^2}(f_2 + \frac{1}{r_2^2} f_3) + f_1$; a = angular position of shaft of helical resistor.

If the gear ratios r_1 and r_2 can be made sufficiently large, the reflected inertia and friction becomes negligibly small compared to J_1 and f_1. The values of r_1 and r_2 depend on the desired rotational speed of the bridge resistors and the speed-torque characteristics of the motor selected. The rotational speed of the bridge resistors in turn depends on the required speed of response and the amplitude of the input variations or output disturbances which are to be followed or regulated. From the standpoint of regulation of output disturbances it is desirable to make the time constant τ_3 much smaller than the time constants of the generator and the magnet. This should be done in order to limit the phase shift of the overall transfer function within the bandwidth of the low-frequency loop, and hence to permit a larger value of the loop gain K and a greater precision of regulation. If the time constant τ_3 is of the order of 1/10 of the predominant time constant of the compensated loop, the phase shift introduced by the servo-power amplifier and control motor is negligible.

It will be seen in subsequent discussion that the predominant time constant of the low-frequency loop is of the order of 5 seconds, hence the effective time constant of the minor loop must be $\leq$ 0.5 second, which implies a bandwidth for $K_m G_m(s)$ of about 13 radians/sec. This provides one basis for evaluating the gear ratio $r_1 r_2$ since the ratio must give the required bandwidth for small amplitude signals, while it must simultaneously permit the magnet current to be changed from zero to maximum in a reasonable time.

A current change from zero to maximum corresponds to 60 revolutions of the bridge shaft. If this change is to occur, say, in 20 seconds, the speed of the bridge shaft must be around 180 rpm. Since the usable maximum speed of the prospective control motor is about 2000 rpm (55 percent of synchronous speed), a gear ratio $r_1 r_2$ of about 11 is indicated.

For a sinusoidal input the position of the bridge shaft may be expressed by

A sin ωt. The maximum velocity of the shaft is therefore $A\omega$ rad/s and the maximum acceleration is $A\omega^2$ rad/s^2. For a bandwidth of 15 rad/s the velocity of the bridge shaft must therefore be $\geqslant$ 15 A rad/s and the acceleration equal to or greater than 225 A rad/s^2. The value of A depends on the amplitude of signal to be followed. A change in magnet current of about 1.0 amp. results from a change of one radian in the bridge-shaft position. Since the expected maximum amplitude of current fluctuation is 2 amp., A is equal to 2. The motor-shaft velocity $\dot{a}$ is related to the bridge-shaft velocity $\dot{a}_b$ as follows:

$$\begin{aligned} \dot{a} &= r_1 r_2 \dot{a}_b \quad , \\ \ddot{a} &= r_1 r_2 \ddot{a}_b \quad . \end{aligned} \tag{11}$$

Therefore for a bridge-shaft acceleration of 225 A = 450 rad/s^2, the control motor-shaft acceleration is $450 r_1 r_2$ rad/s^2 or 4950 if the value $r_1 r_2 = 11$ is used.

The motor selected to meet these requirements was a Diehl type FPF-49-5 with the following specifications:

Voltage	110	Rating, 20 watts at 2200 rpm
Frequency	60 cps	Blower - cooled
Phases	2	Rotor inertia 0.66 oz-in^2
Poles	2	
Synch. speed 3600 rpm		

At zero speed and with 115 v. on each winding, the power input to the control winding of the motor is 60 and to the reference winding is 30 watts.

The internal frictional damping constant f_1 of the motor may be determined from the speed-torque characteristic of the motor and is the slope of the speed-torque curve at the zero-speed point. It was found to be f_1 = 0.032 oz-in/(rad/sec). The friction of the bearings and gear train of the bridge (f_2 and f_3) is negligibly small. The friction of the brushes in the bridge resistors is coulomb-type friction and is therefore not included in the equations of motion of the bridge assembly, but nonetheless must be considered in choosing the proper control

motor. The effect of coulomb friction in the system is essentially the introduction of a small "dead space" about the equilibrium position. However, if the motor has a sufficient excess of torque or acceleration available over the value derived from calculation, the effect of the coulomb friction becomes negligibly small. Also, the coulomb friction itself can be made small by careful mechanical adjustment of the brush carrier mechanism.

An estimate was made of the value of inertia of the load on the motor by determining the inertias of the various gears, bridge-resistor rotors, and tachometer rotor and referring the quantities to the motor shaft. The estimated value of the load inertia was $J_{load} \doteq 10$ oz-in^2. Referred to the control motor shaft this becomes $J_{eff} \doteq 0.08$ oz-in^2. The estimated value of the time constant τ_3 is therefore

$$\tau_3 = \frac{J_o}{f_o} = \frac{0.6 + 0.08}{0.032(386)} = 0.055 \text{ sec.} \tag{12}$$

The maximum acceleration of the motor with load is equal to the torque to inertia ratio, and was calculated to be

$$\frac{\text{rated full load stall torque}}{\text{effective inertia}} = \frac{20(386)}{0.68} = 11,300 \text{ rad/s}^2.$$

This may be seen to meet the requirements with a safety factor of ca. 2, which is ample allowance for the coulomb friction which was neglected in the calculation.

After selection of the control motor it is possible to design the power amplifier to be used to drive it. The circuit used eliminates the need for a high voltage d. c. power supply and offers advantages of simplicity and economy. It is depicted in unit 4 on the system schematic diagram (Fig. 9).

By Fourier analysis the plate current may be written

$$I_p = I_{max}\left(\frac{1}{\pi} + \frac{1}{2}\cos\omega t + \frac{2}{3\pi}\cos 2\omega t + \ldots\right) \quad . \tag{13}$$

The load impedance Z_L is tuned to the fundamental frequency of 60 cps, hence only the fundamental component of the current need be considered. We can thus

write

$$I_p \simeq I_{max} \frac{1}{2} \cos \omega t \tag{14}$$

or

$$I_{prms} = \frac{I_{max}}{2\sqrt{2}} \quad . \tag{15}$$

I_{max} can be determined graphically from the plate characteristics curves for the tube type selected, and I_{prms} then calculated from Eq. (15). The power output of the tube may then be computed from the familiar relation

$$I^2_{prms} R_L = \text{power output} \quad , \tag{16}$$

where R_L is the load resistance selected for the tube. Choice of a suitable type of the tube is thus a matter of selecting an economical and convenient type which will give the required power output to drive the control motor as load. The power required to drive the FPF-49-5 motor to rated output is about 60 watts.

Considerations of economy, convenience of circuitry, and simplicity of future maintenance led to the choice of type 8005 tubes for the output stage of the servo amplifier.

For a plate voltage of E_{pmax} = 1750 v. or E_{prms} = 1250, bias of -70 v., peak grid-signal voltage of 80 v. and a load resistance R_L = 3600Ω, peak plate i_p is found graphically to be 0.425 amp. Thus from Eq. (15) I_{prms} = 0.151 amp. and the power output of the tube is $I^2_{rms} R_L$ = 82 w. which is sufficient to drive the control motor to 133 percent of its rated output. A reserve of power is desirable in the servo amplifier to take advantage of the reserve capacity of the motor. The control-motor output is linear to about 125 percent of rated control voltage and will provide a correspondingly higher power output for short periods without overheating.

Impedance matching of the amplifier to the control motor is complicated somewhat by the variation of the impedance of the motor with output speed, and with variation of the control-winding voltage. However, if the matching is made

for conditions of low speed and of rated control-winding voltage the system will give optimum performance under the conditions requiring the maximum output of the amplifier and control motor. From brake tests made on the control motor the apparent impedance of the control winding under these conditions was found to be $Z_m = (150 + j90)\Omega$ and was nearly constant for zero speed over a range of control voltage from 75 to 115 v. The circuit of the amplifier-load impedance Z_L is shown in Fig. 6. The secondary circuit of the output transformer is tuned to 60 cps by the capacitor 4C36 which balances out the inductive component of the motor impedance. Thus the secondary load is a resistance of 150Ω. Since the coefficient of coupling of the output transformer is nearly unity and resistances of the primary and secondary windings are negligible compared to the winding reactances, the apparent impedance to the right of terminals x, y is

$$Z_{xy} \simeq a^2 Z_m \quad , \qquad (17)$$

where $a = \frac{n_p}{n_s}$ = turns ratio of output transformer 4 T 7. For proper matching $Z_{xy} = R_L = 3600\,\Omega$, therefore $a^2 = 24.1$, or the turns ratio a is 4.91. The value of the capacitor 4C35 was determined experimentally.

$k_2 k_3$ may now be evaluated. For a power output of 60 w. the rms plate current is $I_{prms} = (60/3600)^{1/2} = 0.129$ amp. From Eq. (15) this corresponds to

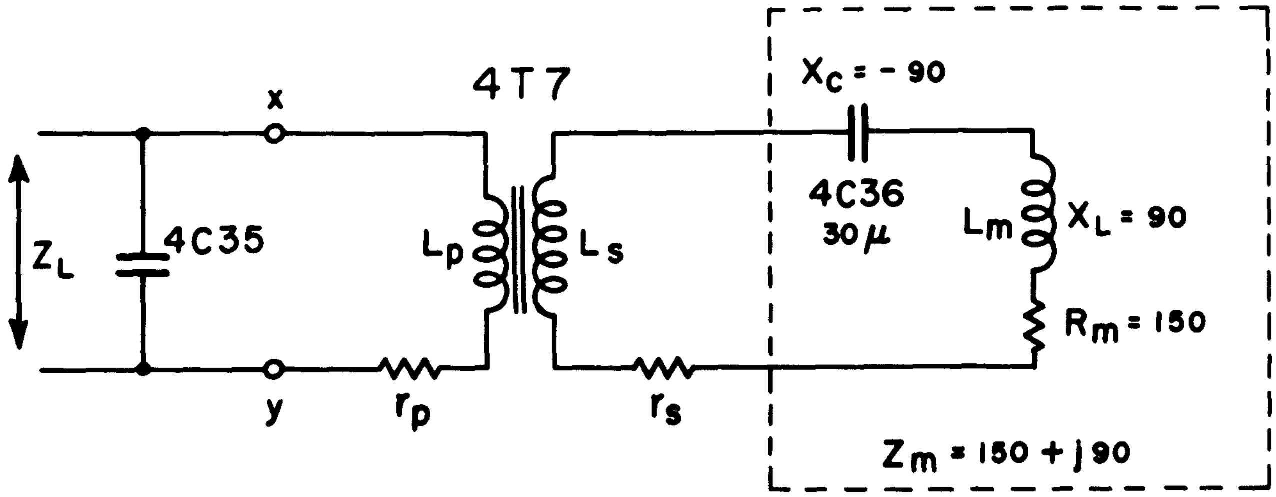


Fig. 6. Output circuit of the servo amplifier.

$$I_{p\ max} = 2\sqrt{2}\,(0.129) = 0.364 \text{ amp.}$$

From the tube data curves this value of $I_{p\ max}$ occurs for a grid signal of 130 v. peak or 92 v. rms. Assuming linearity (only approximately true) the ratio of motor torque to signal voltage of the amplifier output stage is

$$k_t = \frac{\text{stall torque at rated full voltage}}{e_{s_{rms}}} = 20/92 = 0.217 \text{ oz-in/v. rms} \ .$$

The gain of the servo-power amplifier is $k = \left|\frac{e_2}{e_1}\right| = 376$. Therefore $k_2 k_3 = kk_t/f_o = \frac{0.217\,(376)}{0.035} = 2330$ (rad/s)/v.

<u>The generator field circuit.</u> The circuit of the precision helical-resistor bridge and the generator field is schematically shown in simplified form in Fig. 7.

$$\left.\begin{aligned} R_1 + R_2 &= R_3 + R_4 = R_T = 22\,\Omega \\ R_1 = R_4 &= \frac{R_T}{2}\left(1 - \frac{a}{a_o}\right) = \frac{R_T}{2}(1 - \beta) \\ R_3 = R_2 &= \frac{R_T}{2}\left(1 - \frac{a}{a_o}\right) = \frac{R_T}{2}(1 + \beta) \\ -1 &\leqslant \beta \leqslant +1 \end{aligned}\right\} \qquad (18)$$

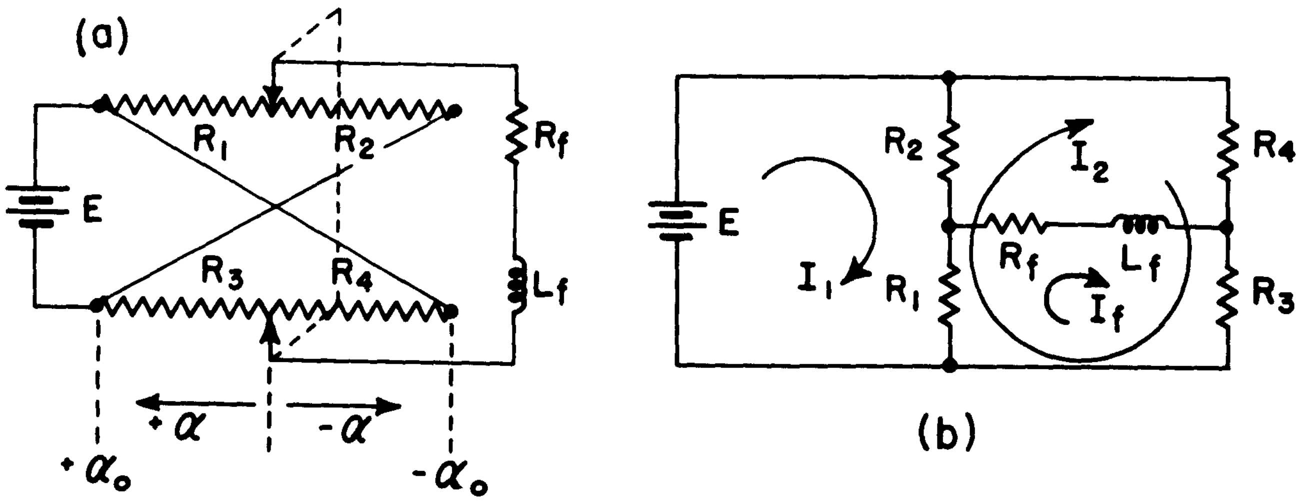


Fig. 7. Schematic diagram of the helical-resistor bridge and the generator-field circuit. a = angular position of bridge-resistor shaft relative to center position; $\pm a_o$ = maximum angular position in either direction of rotation; $-a_o \leqslant a \leqslant +a_o$; E = 100 v. d.c. ; L_f = field inductance = 9.9 hy for step-current change; R_f = field resistance = 37 Ω.

The loop L-transform equations for the circuit are

$$\left.\begin{aligned} (R_2 + R_1)\, I_1(s) - (R_2 + R_1)\, I_2(s) - R_1 I_f(s) &= E \\ -(R_1 + R_2)\, I_1(s) + (R_1 + R_2 + R_3 + R_4)\, I_2(s) - (R_1 + R_3)\, I_f(s) &= 0 \\ -R_1\, I_1(s) + (R_1 + R_3)\, I_2(s) + (R_1 + R_f + R_f + sL_f)\, I_f(s) &= 0 \end{aligned}\right\} \quad . \qquad (19)$$

Equations (19) may be rewritten

$$\begin{aligned} R_T I_1(s) - R_T I_2(s) - \frac{R_T}{2}\left(1 - \beta(s)\right) I_f(s) &= E \\ -R_T\, I_1(s) + 2R_T\, I_2(s) + R_T\, I_f(s) &= 0 \\ -\frac{R_T}{2}\left(1 - \beta(s)\right) I_1(s) + R_T\, I_2(s) + (R_T + R_f + sL_f)\, I_f(s) &= 0 \quad . \end{aligned} \qquad (20)$$

Solution of Eqs. (2) for $I_f(s)$ in terms of $\beta(s)$ yields

$$I_f(s) = \frac{E\,\beta(s)}{\frac{R_T}{2}\left[\beta(s)\right]^2 - \left(\frac{R_T}{2} + R_f + sL_f\right)} \quad . \qquad (21)$$

At all times $0 \leq |\beta| \leq 1$. For $|\beta|$ very small, Eq. (21) becomes

$$I_f(s) = \frac{-E/\left(\frac{R_T}{2} + R_f\right)}{\left(\frac{L_f}{\frac{R_T}{2} + R_f}\right) s + 1} \quad , \qquad (22)$$

and for $|\beta| \rightarrow 1$ it becomes

$$I_f(s) = \frac{-E/R_f}{\left(\frac{L_f}{R_f}\right) s + 1} \quad . \qquad (23)$$

That is, over the entire range of $|\beta|$ I_f is of form

$$I_f(s) = \frac{K_4}{\tau_4 s + 1} \quad , \qquad (24)$$

where τ_4 varies from $\dfrac{L_f}{\frac{R_T}{2} + R_f}$ to $\dfrac{L_f}{R_f}$, and K varies from $\dfrac{E}{\frac{R_T}{2} + R_f}$ to $\dfrac{E}{R_f}$ as $|\beta|$ goes from 0 to 1. In other words, the sensitivity rises from 2.08 to 2.7 and the calculated τ_4 shifts from ca. 0.206 to 0.268. This increase of 30 percent

Table 1. Low-frequency-loop constants.

k_3 = 1060 (rad/sec.)/amp.	τ_3 = 0.055 sec.
k_2k_3 = 2330 (rad/sec.)/v.	
$k_4 = 5.5x(10^{-3})$ amp./rad	τ_4 = 0.2 sec. (calculated) = 0.17 sec. (measured by frequency response test)
k_5 = 15 v./amp.	
k_6 = 13.4 amp./v.	$\tau_6 \simeq$ 7 sec. (for large fluctuations in I_M, say, larger than 2 or 3 amp.) $\simeq 250\mu$ sec. for small fluctuations*

* The large value of τ_6 corresponds to an inductance in the magnet circuit of the order of 0.5 henry, while the small value corresponds to an incremental inductance of the order of 50μ henrys. This wide variation depending on the amplitude of current fluctuation results in a wide variation in the bandwidth of the loop and is one of the reasons for including the minor loop K_8G_8 in the system. Essentially K_8G_8 acts as a low-pass filter, preventing the system from hunting while trying to compensate small rapid fluctuations of the magnet current, and yet permitting high forward loop sensitivity for precise stabilization of the d.c. value of the magnet current.

in sensitivity is not serious from a stability standpoint, but it tends to reduce the overall precision of stabilization obtainable at low levels of magnet current. It turns out, however, that even with this variation in sensitivity with d.c. level of magnet current (or d.c. component of $|\beta|$), sufficient sensitivity is available so that d.c. drift of the magnet current can still be stabilized to the desired precision at low d.c. levels as well as high.

A frequency response test was made on the bridge and field circuit for small $|\beta|$ and the experimental value of τ_4 was 0.17 sec. as compared with 0.207 sec. calculated.

For small $|\beta|$, $K_4 = \frac{2.08}{2\pi 60} = 5.52(10^{-3})$ amp./rad; and for $|\beta| \rightarrow 1$ $K_4 = \frac{2.7}{2\pi 60} = 7.17(10^{-3})$ amp./rad.

The values of the various constants of the low-frequency loop are assembled in Table 1.

Overall response of the low-frequency loop.* Before calculating the overall response of the low-frequency loop, we must first return to the minor loop containing the control motor and calculate the response $\alpha(s)/B(s)$ and the tachometer sensitivity k_7. The expression for $\alpha(s)/B(s)$ may be written

$$\frac{\alpha(s)}{B(s)} = \frac{1}{k_7G_7} \cdot \frac{k_2k_3k_7G_3G_7}{1 + k_2k_3k_7G_3G_7} \quad . \tag{25}$$

With the substitution of the expressions for $G_3(s)$ and $G_7(s)$ and after some manipulation, Eq. (25) becomes

$$\frac{\alpha(s)}{B(s)} = \frac{k_2k_3}{1 + k_2k_3k_7} \cdot \left\{ \frac{1}{s\left[\frac{1}{1 + k_2k_3k_7} s + 1\right]} \right\} \quad . \tag{26}$$

Therefore tachometric feedback serves to introduce kinetic damping by reducing τ_3 by the factor $(1 + k_2k_3k_7)^{-1}$. Since $\tau_3 = J_o/f_o$, the reducing factor in effect increases the friction f_o. As mentioned earlier, bandwidth considerations dictate only that τ_3 be equal to or less than about 0.5 sec., and the unreduced τ_3 clearly satisfies this requirement since it is equal to 0.055 sec. However, for a large step input the control motor must not drive the resistor bridge faster than a rate corresponding to the rate of current increase in the magnet circuit which is fixed by the time constant of the magnet circuit. It is therefore desirable to increase the effective viscous damping of the control motor and thus limit its maximum steady-state velocity. Introduction of tachometric feedback accomplishes this purpose and at the same time provides the incidental benefit of reducing the

* Throughout this discussion, as well as the balance of this report, transfer functions are written in the form KG, where it is understood that G is a function of the complex frequency s, i.e., G = G(s), and K is frequency independent.

effective time constant of the servo-amplifier-control-motor section of the system. Reduction of the time constant, or, alternatively expressed, widening of the bandwidth contributes to increased stability of the system.

An Elinco rotation indicator, Type B-68, was found to be conveniently applicable as the tachometer.* It had a sensitivity k_7 of 0.005 v./(rad/sec.), which was more than sufficient to limit the maximum speed of the control motor to the necessary value. Since k_2k_3 = 2330 (rad/sec.)/v. the term $k_2k_3k_7$ = 11.65 and the reduced time constant τ_3' is

$$\tau_3' = \frac{\tau_3}{1 + k_2k_3k_7} = \frac{0.055}{1 + 11.65} = 0.0044 \text{ sec.}$$

In the actual construction of the system, k_7 was made adjustable to permit its being set to an optimum value during operation of the system.

The gain k_2k_3 of the minor loop, as well as τ_3, is also reduced by the factor $(1 + k_2k_3k_7)^{-1}$, but within limits this loss can be made up by increasing the gain k_1 of the servo-voltage amplifier. The limits of allowable increase in gain are imposed by residual noise level in the tachometer device itself; however, with the tachometer selected, the zero speed residual noise did not cause serious difficulty.

We may now write the expression for the overall transfer function k_oG_o of the forward path of the low-frequency loop

$$k_oG_o = k_1k_mk_4k_5k_6G_mG_4G_6 \quad , \qquad (27)$$

where k_o = 104 μamp./v.-sec. The block diagram of Fig. 3 has been simplified and redrawn in Fig. 8.

Standard graphical-logarithmic, or log-modulus techniques were used to determine the overall closed-loop response, $I_M(j\omega)/\theta_i(j\omega)$.[15] The analytical

* Elinco midget rotation indicator, 110 v., 60 cps, single phase, Type B-68, manufactured by the Electric Indicator Co., Stamford, Conn.

15) For a treatment of log-modulus techniques of servomechanism analysis and synthesis see G. S. Brown and D. P. Campbell, "Principles of Servomechanism," Wiley, New York, 1948; or H. M. James and R. S. Phillips, "Theory of Servomechanisms," McGraw-Hill Book Co., New York, 1947, p. 134.

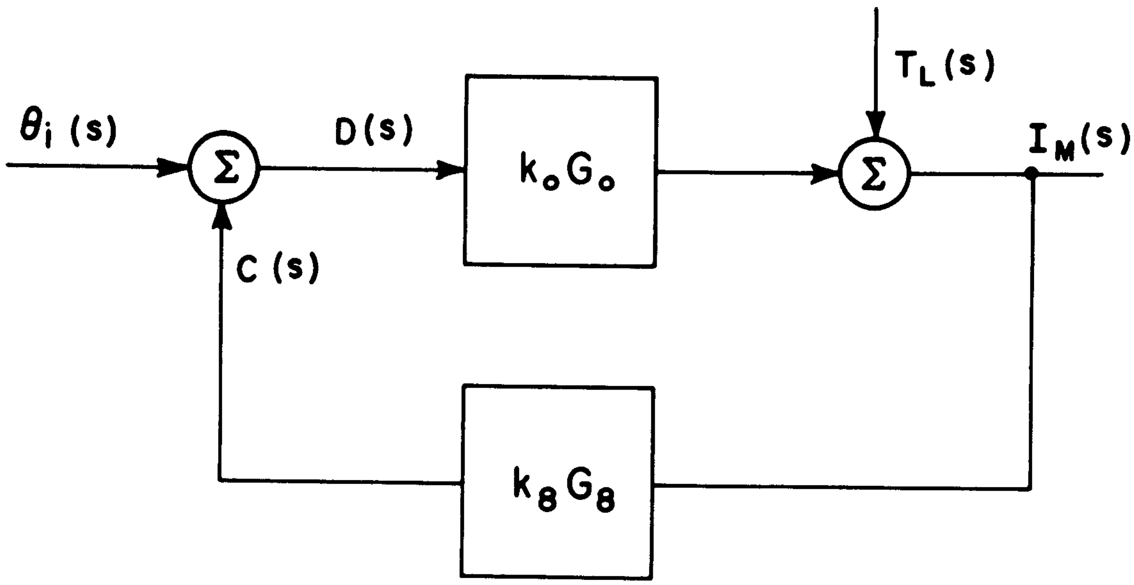


Fig. 8. Simplified block diagram of the low-frequency loop.

$k_o G_o = k_o / s(\tau_1 s + 1)(\tau_3' s + 1)(\tau_4 s + 1)(\tau_6 s + 1)$;

$k_8 G_8 \simeq sM$ for $|\epsilon| < |\epsilon_o|$ in frequency range of interest

$= 0$ for $|\epsilon| > |\epsilon_o|$.

expression for the closed-loop response with $T_L = 0$ is given by the expression

$$\frac{I_M}{\theta_i}(j\omega) = \frac{1}{k_8 G_8(j\omega)} \cdot \frac{k_o G_o(j\omega) k_8 G_8(j\omega)}{1 + k_o G_o(j\omega) k_8 G_8(j\omega)} \quad . \tag{28}$$

To evaluate Eq. (28) by graphical-logarithmic methods, the quantity $G_o(j\omega)\, G_8(j\omega)$ is written in polar form and the log-modulus (Lm) plotted vs. frequency ω and the phase angle (Ang) plotted vs. ω. That is, Lm $G_o(j\omega)\, G_8(j\omega)$ represents 20 $\log_{10} |G_o(j\omega)\, G_8(j\omega)|$ and Ang $G_o(j\omega)\, G_8(j\omega)$ represents the phase angle of the argument. The resulting curves are then combined into one curve as Lm vs. Ang. The gain $k_o k_8$ is calculated by observing the tangency of the $G_o(j\omega)\, G_8(j\omega)$ line with an

appropriate M_p curve, where $M_p = \text{Lm} \frac{I_M}{\theta_i}(j\omega)_{max}$. By the application of a transparency on which families of constant amplitude and constant phase curves are plotted to the same scale as the Lm vs. Ang plot the quantity $\frac{k_o G_o(j\omega) k_8 G_8(j\omega)}{1 + k_o G_o(j\omega) k_8 G_8(j\omega)}$ may be transferred to the original plots of Lm vs. ω and Ang vs. ω. Graphical combination of $\left[k_8 G_8(j\omega)\right]^{-1}$ with this curve yields the overall response of the loop, $I_M(j\omega)/\theta_i(j\omega)$.

Selection of the appropriate value of M_p depends on the required band width of the loop, the value of loop gain required by the regulation specifications, and by the amount of resonant peak permissible in the response characteristic. The last consideration is related closely to the required transient response of the system.

We have seen that we wish the band width of the low-frequency loop to be of the order of a few cps if possible, and that we must not have any overshoot in the transient response. This latter requirement implies that the frequency response of the loop should not have any resonant peak. The amount of loop gain required may be determined by considering the regulatory behavior of the loop.

It can be shown that the regulatory behavior of the system is related to the behavior as a servomechanism by the following relation

$$\left[\frac{D(j\omega)}{T_L(j\omega)}\right]_{\theta_i = 0} = \left[\frac{k_8 G_8(j\omega)}{k_o G_o(j\omega)} \cdot \frac{I_M(j\omega)}{\theta_i(j\omega)}\right]_{T_L = 0} \qquad (29)$$

Therefore

$$\left[\frac{D(j\omega)}{T_L(j\omega)}\right]_{\theta_i = 0} = \frac{k_8 G_8(j\omega)}{1 + k_o G_o(j\omega) k_8 G_8(j\omega)} \qquad (30)$$

$D(j\omega)/T_L(j\omega)$ may be plotted to determine if the specifications on precision of regulation are met throughout the band width of the loop.

On the basis of several trial calculations, the value of M_p selected was 6 db. The value of loop gain was found to be $k_o k_8 = 21.9$ db. Since $k_8 = -60$ db it follows

that k_o = 81.9 db or 8920 amp./v.-sec. The required value of k_1 is therefore $k_o/104$ = 85.7. In the actual construction of the system, however, k_1 was made adjustable to permit selection of an optimum value during operation of the system.

The High-Frequency Loop

The required band width of the high-frequency loop is determined primarily by the required precision of stabilization and by the inherent filtering action of the magnet circuit inductance. An analysis of the noise present in the unregulated magnet current revealed that for precision of stabilization of the order of one part in 10^5 a bandwidth of about 12 kc/s is required. This, at first glance, may seem surprising in view of the fact that the magnet inductance for large current changes is of the order of 0.5 hy, but not when it is remembered that for small current fluctuations the apparent inductance of the magnet is of the order of only 50 μhy. Above about 12 kc/s the filtering action of the magnet circuit reduces the remaining noise components to less than the acceptable limit of one part in 10^5, or $<$ ca. 4 ma.

An experimental model of a circuit for the high-frequency loop was constructed along the lines of the system described and built by Sommers,[11)] but proved unsatisfactory in this application. It is possible that with extensive modification and refinement it could have been made to work satisfactorily, but since for present uses of the magnet the existing one-channel control system seems adequate, it has not seemed expedient at this time to expend further effort in development of the high-frequency loop. Future experiments utilizing the magnet and its control system will probably require greater precision of field stabilization, and addition of the second channel to compensate the remaining noise fluctuations will then be necessary. This can be accomplished by one of at least two methods: first, by a dynamic electronic filter with a bandpass from 1 cps to 12 kc/s driving the magnet in parallel with the generator; and second, by means

of small auxilliary coils at the magnet air gap which would be driven electronically as demanded by an error-measuring device placed in the gap itself. The latter method lends itself to the use of a rotating-coil gaussmeter as the error-sensing element, or to a nuclear resonance-sensing element. Either method can be expected to achieve an overall precision of regulation of the order of one part in 10^5 when properly matched frequency-response-wise to the existing low-frequency channel.

Conclusions

The present one-channel control system has been designed and constructed to compensate for the drift and small shifts in the d.c. value of the current at any desired current level as well as to follow-up relatively infrequent step changes in the input command, but does not compensate for the components of noise whose frequency is greater than ca. 1 cps. That is, it essentially determines the set point or d.c. value of the magnet current. Slow thermal drift and small shifts in the d.c. value are held to within one part in 10^5. Superimposed on this stabilized d.c. value remains the noise whose frequency spectrum lies above 1 cps. The amplitude of this noise is less than 0.5 percent of the d.c. value. Future refinement of the system calls for an additional channel whose function is to compensate for the remaining noise fluctuations. However, as pointed out above, for present uses of the magnet the existing one-channel control system has proved quite adequate.

Features of the present control system include continuous variability of the magnet current from maximum negative to maximum positive values with stabilization provided at any desired level of the current, quick reversibility of the polarity for convenience in demagnetization, and complete elimination of switching in the magnet circuit itself, thus avoiding voltage-surges due to interrupting the heavy currents in the inductive magnet circuit and avoiding arcing or insulation breakdown.

A conclusion reached during development of this control system should be mentioned briefly: one cannot overemphasize the importance of treating a proposed magnet and associated control system as an integrated problem. The present system was developed in two more or less distinct phases. The needs of this laboratory initially were for a large electromagnet to provide magnetic fields of the order of 18,000 gauss in order to facilitate certain experimental research which was in progress at the time. It was desired to complete the magnet as quickly as possible. Precise stabilization of the fields produced by the proposed magnet was not required by the contemplated experiments which would use the magnet, and it was therefore decided to design and construct the magnet and its power source first and defer development of a stabilizing system until a later date if and when it were required.

From considerations of efficient use of copper in the coil windings and economical utilization of available power sources it was decided to make the magnet of the low-impedance, high-current type. Such a design permits more copper to be placed in a given winding cross section and thus permits development of more ampere-turns magnetizing force for a given winding volume than is possible for a high-impedance winding in which more of the available space is necessarily taken by the insulation of the winding. Furthermore, the low-impedance design lends itself well to simple, efficient designs of cooling systems, an important consideration when designing a large electromagnet.

Subsequent to the construction of the magnet and its use in the original research experiments, new experiments dictated the need for stabilization of the field and therefore development of a suitable automatic regulating system was initiated and completed as recorded in this report. It was found in the development of this sytem that, although the low-impedance type of magnet design had offered distinct advantages in efficient use of copper and effective cooling, it had

greatly complicated the problem of satisfactory stabilization. If the entire system were to be designed as a unit, it is probable that a high-impedance design should be used instead. Various schemes of electronic control may be applied conveniently to a high-impedance magnet which are not at all possible for the other type. For an example, it is possible to use an electronically regulated three-phase rectifier supply directly from the 220 v. 3-phase bus. Such a system has been satisfactorily applied to the large electromagnet used in the High Voltage Laboratory of this institute, although the system there used would not be applicable for the uses in this laboratory without extensive revisions and modifications. For one thing, it makes no provision for continuous variability of the magnet field from negative to positive values. Nonetheless, such a system or some other electronic system could probably be devised to accomplish the required stabilization and control if the magnet were a high-impedance device, and do the job considerably more simply than was possible in the present system. It is clear, therefore, that in the long run it would be wise to treat a proposed magnet and its control system as one integral problem in development, even though this implies a much greater initial commitment of laboratory time and funds.

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